Topic: Implicit differentiation, second derivatives

Question: Use implicit differentiation to find the second derivative.

$$x^3 + y^3 = 9$$

Answer choices:

A $$y'' = -2x\left(\frac{y^3 - x^3}{y^6}\right)$$

B $$y'' = -2x\left(\frac{y^3 + x^3}{y^5}\right)$$

C $$y'' = -2x\left(\frac{y^3 - x^3}{y^5}\right)$$

D $$y'' = -2x\left(\frac{y^3 + x^3}{y^6}\right)$$

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Solution: B

To find the second derivative of a function we need to start by calculating the first derivative, and then take the derivative of the first derivative to find the second derivative.

Using implicit differentiation to take the derivative of our function, we get

$$3x^2 + 3y^2y' = 0$$

Solving for y', we get

$$3y^2y' = -3x^2$$

$$y' = \frac{-3x^2}{3y^2}$$

$$y' = \frac{-x^2}{y^2}$$

Now we'll take the derivative of y' to find y''.

$$y'' = \frac{(-2x)\left(y^2\right) - \left(-x^2\right)\left(2yy'\right)}{\left(y^2\right)^2}$$

$$y'' = \frac{-2xy^2 + 2x^2yy'}{y^4}$$

$$y'' = \frac{2xy\left(-y + xy'\right)}{y^4}$$

$$y'' = \frac{2x(-y + xy')}{y^3}$$

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We can simplify further by plugging in for y' using $y' = \frac{-x^2}{y^2}$.

$$y'' = \frac{2x\left[-y + x\left(\frac{-x^2}{y^2}\right)\right]}{y^3}$$

$$y'' = \frac{2x\left(-y - \frac{x^3}{y^2}\right)}{y^3}$$

$$y'' = \frac{2x\left(\frac{-y^3 - x^3}{y^2}\right)}{y^3}$$

$$y'' = 2x\left(\frac{-y^3 - x^3}{y^5}\right)$$

$$y'' = -2x\left(\frac{y^3 + x^3}{y^5}\right)$$

The second derivative of

$$x^3 + y^3 = 9$$

is

$$y'' = -2x\left(\frac{y^3 + x^3}{y^5}\right)$$

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Topic: Implicit differentiation, second derivatives

Question: Use implicit differentiation to find the second derivative.

$$x^2 + 2xy + y^3 = 25$$

Answer choices:

A $$y'' = \frac{-2\left(9y^4 + 24xy^2 - 8xy + 12x^2y - 4x^2\right)}{\left(2x + 3y^2\right)\left(4x^2 + 12xy^2 + 9y^4\right)}$$

B $$y'' = \frac{-2\left(9y^4 + 24xy^2 + 8xy + 12x^2y + 4x^2\right)}{\left(2x + 3y^2\right)\left(4x^2 + 12xy^2 + 9y^4\right)}$$

C $$y'' = \frac{-2\left(-9y^4 - 24xy^2 - 8xy - 12x^2y - 4x^2\right)}{\left(2x + 3y^2\right)\left(4x^2 + 12xy^2 + 9y^4\right)}$$

D $$y'' = \frac{-2\left(9y^4 + 24xy^2 - 8xy + 12x^2y - 4x^2\right)}{\left(2x - 3y^2\right)\left(4x^2 + 12xy^2 + 9y^4\right)}$$

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Solution: A

To find the second derivative of a function we need to start by calculating the first derivative, and then take the derivative of the first derivative to find the second derivative.

Using implicit differentiation to take the derivative of our function, plus product rule for the middle term on the left, we get

$$2x + \left[(2)(y) + (2x)(1)(y')\right] + 3y^2y' = 0$$

$$2x + 2y + 2xy' + 3y^2y' = 0$$

Solving for y', we get

$$2xy' + 3y^2y' = -2x - 2y$$

$$y'\left(2x + 3y^2\right) = -2x - 2y$$

$$y' = \frac{-2x - 2y}{2x + 3y^2}$$

Now we'll use quotient rule to take the derivative of y' and find y''.

$$y'' = \frac{\left[-2 - 2(1)(y')\right]\left(2x + 3y^2\right) - (-2x - 2y)(2 + 6yy')}{\left(2x + 3y^2\right)^2}$$

$$y'' = \frac{-4x - 6y^2 - 4xy' - 6y^2y' - \left(-4x - 12xyy' - 4y - 12y^2y'\right)}{4x^2 + 12xy^2 + 9y^4}$$

$$y'' = \frac{-4x - 6y^2 - 4xy' - 6y^2y' + 4x + 12xyy' + 4y + 12y^2y'}{4x^2 + 12xy^2 + 9y^4}$$

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$$y'' = \frac{-6y^2 - 4xy' + 12xyy' + 4y + 6y^2y'}{4x^2 + 12xy^2 + 9y^4}$$

$$y'' = \frac{-6y^2 + 4y + y'\left(-4x + 12xy + 6y^2\right)}{4x^2 + 12xy^2 + 9y^4}$$

We can simplify further by plugging in for y' using $y' = \frac{-2x - 2y}{2x + 3y^2}$.

$$y'' = \frac{-6y^2 + 4y + \left(\frac{-2x - 2y}{2x + 3y^2}\right)\left(-4x + 12xy + 6y^2\right)}{4x^2 + 12xy^2 + 9y^4}$$

$$y'' = \frac{-6y^2 + 4y + \frac{8x^2 + 8xy}{2x + 3y^2} + \frac{-24x^2y - 24xy^2}{2x + 3y^2} + \frac{-12xy^2 - 12y^3}{2x + 3y^2}}{4x^2 + 12xy^2 + 9y^4}$$

$$y'' = \frac{\frac{-12xy^2 - 18y^4}{2x + 3y^2} + \frac{8xy + 12y^3}{2x + 3y^2} + \frac{8x^2 + 8xy}{2x + 3y^2} + \frac{-24x^2y - 24xy^2}{2x + 3y^2} + \frac{-12xy^2 - 12y^3}{2x + 3y^2}}{4x^2 + 12xy^2 + 9y^4}$$

$$y'' = \frac{\frac{-12xy^2 - 18y^4 + 8xy + 12y^3 + 8x^2 + 8xy - 24x^2y - 24xy^2 - 12xy^2 - 12y^3}{2x + 3y^2}}{4x^2 + 12xy^2 + 9y^4}$$

$$y'' = \frac{-12xy^2 - 18y^4 + 8xy + 12y^3 + 8x^2 + 8xy - 24x^2y - 24xy^2 - 12xy^2 - 12y^3}{\left(2x + 3y^2\right)\left(4x^2 + 12xy^2 + 9y^4\right)}$$

$$y'' = \frac{-18y^4 - 48xy^2 + 16xy - 24x^2y + 8x^2}{\left(2x + 3y^2\right)\left(4x^2 + 12xy^2 + 9y^4\right)}$$

$$y'' = \frac{-2\left(9y^4 + 24xy^2 - 8xy + 12x^2y - 4x^2\right)}{\left(2x + 3y^2\right)\left(4x^2 + 12xy^2 + 9y^4\right)}$$

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The second derivative of

$$x^2 + 2xy + y^3 = 25$$

is

$$y'' = \frac{-2\left(9y^4 + 24xy^2 - 8xy + 12x^2y - 4x^2\right)}{\left(2x + 3y^2\right)\left(4x^2 + 12xy^2 + 9y^4\right)}$$

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Topic: Implicit differentiation, second derivatives

Question: Use implicit differentiation to find the second derivative.

$$x^2y^2 + 3xy = 100$$

Answer choices:

A $$y'' = \frac{2xy^2 + 3y + \left(2x^2y + 3x\right)\left(4x^2y^3 + 12xy^2 + 9y\right)}{\left(2x^2y + 3x\right)^3}$$

B $$y'' = \frac{\left(-2xy^2 - 3y\right)\left(4x^3y^2 + 12x^2y + 9x\right) + \left(2x^2y + 3x\right)\left(4x^2y^3 + 12xy^2 + 9y\right)}{\left(2x^2y + 3x\right)^2}$$

C $$y'' = \frac{\left(2xy^2 + 3y\right)\left(4x^3y^2 + 12x^2y + 9x\right)}{\left(2x^2y + 3x\right)^2}$$

D $$y'' = \frac{\left(2xy^2 + 3y\right)\left(4x^3y^2 + 12x^2y + 9x\right) + \left(2x^2y + 3x\right)\left(4x^2y^3 + 12xy^2 + 9y\right)}{\left(2x^2y + 3x\right)^3}$$

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Solution: D

To find the second derivative of a function we need to start by calculating the first derivative, and then take the derivative of the first derivative to find the second derivative.

Using implicit differentiation to take the derivative of our function, plus product rule for the first and second terms on the left, we get

$$\left[(2x)\left(y^2\right)+\left(x^2\right)\left(2yy'\right)\right]+\left[(3)(y)+(3x)\left(1y'\right)\right]=0$$

$$2xy^2+2x^2yy'+3y+3xy'=0$$

Solving for y', we get

$$2x^2yy'+3xy'=-2xy^2-3y$$

$$y'\left(2x^2y+3x\right)=-2xy^2-3y$$

$$y'=\frac{-2xy^2-3y}{2x^2y+3x}$$

Now we'll use quotient and product rules to take the derivative of y' and find y''.

$$y''=\frac{\left[(-2)\left(y^2\right)+(-2x)\left(2yy'\right)-3y'\right]\left(2x^2y+3x\right)-\left(-2xy^2-3y\right)\left[(4x)(y)+\left(2x^2\right)\left(1y'\right)+3\right]}{\left(2x^2y+3x\right)^2}$$

$$y''=\frac{\left(-2y^2-4xyy'-3y'\right)\left(2x^2y+3x\right)-\left(-2xy^2-3y\right)\left(4xy+2x^2y'+3\right)}{\left(2x^2y+3x\right)^2}$$

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$$y'' = \frac{-4x^2y^3 - 6xy^2 - 8x^3y^2y' - 12x^2yy' - 6x^2yy' - 9xy' - \left(-8x^2y^3 - 4x^3y^2y' - 6xy^2 - 12xy^2 - 6x^2yy' - 9y\right)}{\left(2x^2y + 3x\right)^2}$$

$$y'' = \frac{-4x^2y^3 - 6xy^2 - 8x^3y^2y' - 12x^2yy' - 6x^2yy' - 9xy' + 8x^2y^3 + 4x^3y^2y' + 6xy^2 + 12xy^2 + 6x^2yy' + 9y}{\left(2x^2y + 3x\right)^2}$$

$$y'' = \frac{-4x^3y^2y' + 4x^2y^3 - 12x^2yy' + 12xy^2 - 9xy' + 9y}{\left(2x^2y + 3x\right)^2}$$

$$y'' = \frac{y'\left(-4x^3y^2 - 12x^2y - 9x\right) + 4x^2y^3 + 12xy^2 + 9y}{\left(2x^2y + 3x\right)^2}$$

We can simplify further by plugging in for y' using $y' = \dfrac{-2xy^2 - 3y}{2x^2y + 3x}$.

$$y'' = \frac{\frac{-2xy^2 - 3y}{2x^2y + 3x}\left(-4x^3y^2 - 12x^2y - 9x\right) + 4x^2y^3 + 12xy^2 + 9y}{\left(2x^2y + 3x\right)^2}$$

$$y'' = \frac{\frac{\left(-2xy^2 - 3y\right)\left(-4x^3y^2 - 12x^2y - 9x\right)}{2x^2y + 3x} + \frac{\left(2x^2y + 3x\right)\left(4x^2y^3 + 12xy^2 + 9y\right)}{2x^2y + 3x}}{\left(2x^2y + 3x\right)^2}$$

$$y'' = \frac{\frac{\left(-2xy^2 - 3y\right)\left(-4x^3y^2 - 12x^2y - 9x\right) + \left(2x^2y + 3x\right)\left(4x^2y^3 + 12xy^2 + 9y\right)}{2x^2y + 3x}}{\left(2x^2y + 3x\right)^2}$$

$$y'' = \frac{\left(-2xy^2 - 3y\right)\left(-4x^3y^2 - 12x^2y - 9x\right) + \left(2x^2y + 3x\right)\left(4x^2y^3 + 12xy^2 + 9y\right)}{\left(2x^2y + 3x\right)^3}$$

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$$y'' = \frac{-\left(2xy^2+3y\right)\left(-4x^3y^2-12x^2y-9x\right)+\left(2x^2y+3x\right)\left(4x^2y^3+12xy^2+9y\right)}{\left(2x^2y+3x\right)^3}$$

$$y'' = \frac{\left(2xy^2+3y\right)\left(4x^3y^2+12x^2y+9x\right)+\left(2x^2y+3x\right)\left(4x^2y^3+12xy^2+9y\right)}{\left(2x^2y+3x\right)^3}$$

The second derivative of $x^2y^2+3xy=100$ is

$$y'' = \frac{\left(2xy^2+3y\right)\left(4x^3y^2+12x^2y+9x\right)+\left(2x^2y+3x\right)\left(4x^2y^3+12xy^2+9y\right)}{\left(2x^2y+3x\right)^3}$$

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